

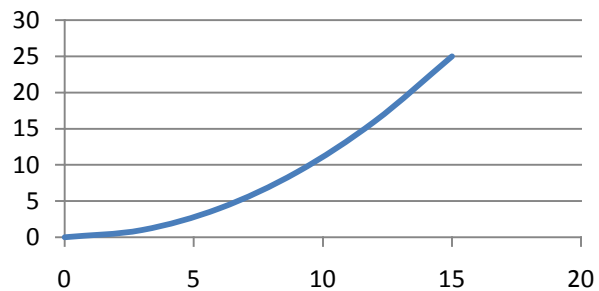
## Parametric Differentiation

Parametric Equations in  $x$  and  $y$  are ones where both  $x$  and  $y$  are defined in terms of a third variable, called a **parameter**. The parameter is often  $t$ , representing time.

Example:  $\begin{cases} x = 3t \\ y = t^2 \end{cases}$  are the parametric equations for an object that starts at the origin at time 0, and its position relative to the  $x$ -axis after  $t$  seconds is given by  $x = 3t$  and its position relative to the  $y$ -axis after  $t$  seconds is given by  $y = t^2$ .

When time,  $t$ , is known, the  $(x, y)$  position can be calculated:

$t$	0	1	2	3	4	5
$x = 3t$	0	3	6	9	12	15
$y = t^2$	0	1	4	9	16	25



We use the Chain Rule to differentiate parametric equations:

$$\text{Chain Rule: } \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \quad \text{or} \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

depending on whether the 3<sup>rd</sup> parameter is  $t$  or  $\theta$ .

**Example 1:** Differentiate the following parametric equation:  $x = 6t^2$ ,  $y = 2t$ .

$$\frac{dy}{dt} = 2 \qquad \frac{dx}{dt} = 12t \qquad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 2 \times \frac{1}{12t} = \frac{2}{12t} = \frac{1}{6t}$$

**Example 2:** The following parametric equation describes a circle:  $x = 2 \cos \theta$ ,  $y = 2 \sin \theta$ .

Differentiate to find  $\frac{dy}{dx}$ .

$$\begin{aligned} \frac{dy}{d\theta} &= 2 \cos \theta & \frac{dx}{d\theta} &= -2 \sin \theta \\ \frac{dy}{dx} &= \frac{dy}{d\theta} \times \frac{d\theta}{dx} = 2 \cos \theta \times \frac{1}{-2 \sin \theta} = \frac{2 \cos \theta}{-2 \sin \theta} = \frac{-\cos \theta}{\sin \theta} = -\cot \theta \end{aligned}$$

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